ASSIGNMENT - 3

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UNIT-3 "MATRICES"

Que.1 Find the normal form of the matrix and hence find its Rank.

$$A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

$C_1\left(\frac{1}{2}\right)$, $C_4\left(\frac{1}{2}\right)$ $\Rightarrow$ $A = \begin{bmatrix} 1 & 1 & 3 & 3 \\ 0 & 3 & 2 & 1 \\ -1 & -1 & -3 & 2 \end{bmatrix}$

$R_{31}(1)$

$$A = \begin{bmatrix} 1 & 1 & 3 & 3 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$C_{21}(-1)$, $C_{31}(-3)$, $C_{41}(-3)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$C_2\left(\frac{1}{3}\right)$, $C_3\left(\frac{1}{2}\right)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$C_{32}(-1)$, $C_{42}(-1)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$C_{34}(-1)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} I_3 & 0 \end{bmatrix}$$

Rank of A $\rightarrow$ $\rho(A) = 3$

Nullity Not defined

Que. 2 Find the Normal form of the following Matrices.

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

Sol.^{n} ⇒ Let A denote the given matrix :-

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

$R_{21}(-1)$, $R_{31}(-2)$, $R_{41}(-4)$

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -3 \\ 1 & -2 & -5 & -8 \end{bmatrix}$$

$R_{12}(-1)$, $R_{42}(-1)$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -3 \\ 0 & -3 & -6 & -9 \end{bmatrix}$$

$R_{21}(-1)$, $R_4(-1/3)$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_{32}(-1)$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_{21}(-2)$, $C_{31}(-3)$, $C_{41}(-4)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2(-1)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_{32}(-2)$, $C_{42}(-3)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

Rank of A $\rightarrow$ $\delta(A) = 2$

Nullity of A $\rightarrow$ $N(A) = 4 - 2$
$= 2$

Que.3 Find that for what values of λ, μ, the equations.

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = 4$$

have (1) No solution
(2) a unique sol.^{n}
(3) Infinite many sol.^{n}

Sol.^{n} $\Rightarrow$ The matrix eq.^{n} $AX = B$ is given by -

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 4 \end{bmatrix}$$

$$\therefore [A \vdots B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & 4 \end{array}\right]$$

$$= \begin{matrix} R_{21}(-1) \\ R_{32}(-1) \end{matrix} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-5 & \mu-10 \end{array}\right]$$

Now consider the following cases:-

- Case-1 - If $\lambda \neq 3$, then $\rho(A) = \rho(A \vdots B) = 3$

Hence in this case the eq.^{n} are consistent and will have a unique sol.^{n}.

- Case-2 - If $\lambda = 3$, $\mu = 10$, then $\rho(A) = \rho[A \vdots B]$ $= 2 < 3$

Hence in this care the eq.^{n} are consistent and will have infinite many solutions.

- Case-3 - If $\lambda = 3$, $\mu \neq 10$ then $\rho[A \vdots B] = 3$, $\rho(A) = 2$
$\therefore \rho(A) \neq \rho[A \vdots B]$

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Que. 4 Find the eigen values and eigen vectors of the matrix.

$$A = \begin{bmatrix} 8 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Character eq.^{n} for matrix

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$6-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 3-\lambda \end{vmatrix} + 2 \begin{vmatrix} -2 & -1 \\ 2 & 3-\lambda \end{vmatrix} + 2 \begin{vmatrix} -2 & 3-\lambda \\ 2 & 1 \end{vmatrix} = 0$$

$$6-\lambda \left[(3-\lambda)^2 - 1 \right] + 2 \left[(-6+2\lambda) + 2 \right] + 2 \left[2 - (6+2\lambda) \right] = 0$$

$$6-\lambda \left[9 + \lambda^2 - 6\lambda - 1 \right] + \left[-12 + 4\lambda + 4 \right] + \left[4 - 12 + 4\lambda \right] = 0$$

$$54 + 6\lambda^2 - 36\lambda - 6 - 9\lambda - \lambda^3 + 6\lambda^2 + \lambda - 12$$

$$+ 4\lambda + 4 + 4 - 12 + 4\lambda = 0$$

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$54-6-12+8-12+6\lambda^2+6\lambda^2-36\lambda-9\lambda+\lambda$
$+8\lambda-\lambda^3=0$

$[48-24+8]+12\lambda^2-45\lambda+9\lambda-\lambda^3=0$

$32+12\lambda^2-36\lambda-\lambda^3=0$

$\lambda^3-12\lambda^2+36\lambda-32=0$

$\therefore$ Eigen value of matrix (A) are 2, 2, 8.

The Eigen vector corresponding the eigen value of 2 $\boxed{\lambda=2}$

$(A-2I)X=0$

$$\begin{bmatrix} 6-2 & -2 & 2 \\ -2 & 3-2 & -1 \\ 2 & -1 & 3-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$$R_1\left(\frac{1}{2}\right) \quad R_{32}(-1) \begin{bmatrix} 2 & -1 & 1 \\ -2 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$R_{21}(1) \begin{bmatrix} 2 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad -(1)$$

$$\Rightarrow 2x_1 - x_2 + x_3 = 0 \quad -(2)$$

Let,

$x_2 = 0$, $x_3 = 2$, then from eq.^{n} (2)

$$2x_1 = -2$$

$$x_1 = -1$$

Again if $x_2 = 2$, $x_3 = 0$, then from eq.^{n} (2)

$$2x_1 = 2$$

$$x_1 = 1$$

Thus two independent eigen vectors for given value $\lambda = 2$ are –

$x_1 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

Again, the eigen vector corresponding the eigen value of 8 :-

$\boxed{\lambda = 8}$

$$(A - 8I) X = 0$$

$$\Rightarrow \begin{bmatrix} 6-8 & -2 & 2 \\ 2 & 3-8 & -1 \\ -2 & -1 & 3-8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2 & -2 & 2 \\ 2 & -5 & -1 \\ -2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\frac{x_1}{\begin{vmatrix} -5 & -1 \\ -1 & -5 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} -2 & -1 \\ 2 & -5 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 2 & -5 \\ -2 & -1 \end{vmatrix}}$$

$$\frac{x_1}{25-1} = \frac{x_2}{10+2} = \frac{x_3}{[-2-10]}$$

$$\frac{x_1}{24} = \frac{x_2}{12} = \frac{x_3}{-12}$$

divided by 12

Hence, $\frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{1}$

$\therefore X_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ is eigen vector corresponding to $\boxed{\lambda = 8}$

<u>Que. 5</u> Find the character root (eigen values) and corresponding characteristics (eigen) vectors of the matrix :-

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & 4 & 3 \end{bmatrix}$$

<u>Sol.^{n}</u> $\Rightarrow$

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & 4 & 3 \end{bmatrix}$$

Characteristics eq.^{n} for matrix A.

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$C_{23}(3) \begin{bmatrix} 8-\lambda & 0 & 2 \\ -6 & -5-\lambda & -4 \\ 2 & 5-3\lambda & 3-\lambda \end{bmatrix} = 0$$

$$8-\lambda \begin{vmatrix} -5-\lambda & -4 \\ 5-3\lambda & 3-\lambda \end{vmatrix} - 0 \begin{vmatrix} \cancel{\;} \end{vmatrix} + 2 \begin{vmatrix} -6 & -5-\lambda \\ 2 & 5-3\lambda \end{vmatrix} = 0$$

$$(8-\lambda)\left[(-5-\lambda)(3-\lambda) - 4(5-3\lambda)\right] + 2\left[(-30+18\lambda) - (-10-2\lambda)\right] = 0$$

$$(8-\lambda)\left[-15+5\lambda-3\lambda+\lambda^2+20-12\lambda\right]$$
$$-60+36\lambda+20+4\lambda = 0$$

$$(8-\lambda)(\lambda^2-10\lambda+5) - 40\lambda - 40 = 0$$

$$8\lambda^2 - 80\lambda + \cancel{40} - \lambda^3 + 10\lambda^2 - 5\lambda + 40\lambda$$
$$\cancel{-40} = 0$$

$$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$$

$$\lambda(-\lambda^2 + 18\lambda - 45) = 0$$

$$\boxed{\lambda = 0, 3, 15}$$

Eigen values of A are 0, 3, 15.

The eigen vector corresponding the eigen value of 0.

$$\lambda = 0$$

$$(A - 0I)X = 0$$

$$\Rightarrow \begin{bmatrix} 8-0 & -6 & 2 \\ -6 & 7-0 & -4 \\ 2 & -4 & 3-0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & 4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Now, $\dfrac{x_1}{\begin{bmatrix} 7 & -4 \\ -4 & 3 \end{bmatrix}} = \dfrac{x_2}{\begin{bmatrix} -6 & -4 \\ 2 & 3 \end{bmatrix}} = \dfrac{x_3}{\begin{bmatrix} -6 & 7 \\ 2 & -4 \end{bmatrix}}$

$$\frac{x_1}{(21-16)} = \frac{x_2}{(-18+8)} = \frac{x_3}{(24-14)}$$

$$\frac{x_1}{5} = \frac{x_2}{-10} = \frac{x_3}{10}$$

divided by 5

Hence, $\dfrac{x_1}{1} = \dfrac{x_2}{-2} = \dfrac{x_3}{2}$

$\therefore x_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$ is the eigen vector for $\lambda = 0$.

Again, The eigen vector corresponding the eigen value of 3 :-

$$\boxed{\lambda = 3}$$

$$(A - 3I)X = 0$$

$$\Rightarrow \begin{bmatrix} 8-3 & -6 & 2 \\ -6 & 7-3 & -4 \\ 2 & -4 & 3-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Now,

$$\frac{x_1}{\begin{bmatrix} 4 & -4 \\ -4 & 0 \end{bmatrix}} = \frac{x_2}{\begin{bmatrix} -6 & -4 \\ 2 & 0 \end{bmatrix}} = \frac{x_3}{\begin{bmatrix} -6 & 4 \\ 2 & -4 \end{bmatrix}}$$

$$\frac{x_1}{[0-16]} = \frac{x_2}{[0+8]} = \frac{x_3}{[24-8]}$$

$$\frac{x_1}{-16} = \frac{x_2}{8} = \frac{x_3}{16}$$

divided by -8

Hence, $\frac{x_1}{2} = \frac{x_2}{1} = \frac{x_3}{-2}$

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$\therefore\ x_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$ is the eigen vector for the eigen value $\lambda = 3$

Again, The eigen vector corresponding the eigen value of 15.

$$\boxed{\lambda = 15}$$

$$(A - 15I)X = 0$$

$$\Rightarrow \begin{bmatrix} 8-15 & -6 & 2 \\ -6 & 7-15 & -4 \\ 2 & -4 & 3-15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Now,

$$\frac{x_1}{\begin{bmatrix} -8 & -4 \\ -4 & -12 \end{bmatrix}} = \frac{x_2}{\begin{bmatrix} -6 & -4 \\ 2 & -12 \end{bmatrix}} = \frac{x_3}{\begin{bmatrix} -6 & -8 \\ 2 & -4 \end{bmatrix}}$$

$$\frac{x_1}{[96-16]} = \frac{x_2}{[72+8]} = \frac{x_3}{[24+16]}$$

$$\frac{x_1}{80} = \frac{x_2}{80} = \frac{x_3}{40}$$

divided by 40,

Hence,

$$\frac{x_1}{2} = \frac{x_2}{2} = \frac{x_3}{1}$$

$\therefore$ $x_3 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ is the eigen vector for the eigen value $\boxed{\lambda = 15}$

<u>Que. 8</u> If Matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ then verify the cayley Hamilton theoram, Hence Find A^{-1}.

<u>Solⁿ.</u> $\Rightarrow$ Given

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

Characteristics eq^n for matrix A.

$$|A-\lambda I| = 0$$

$$A = \begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$1-\lambda \begin{vmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{vmatrix} - 0 + 2 \begin{vmatrix} 0 & 2-\lambda \\ 2 & 0 \end{vmatrix} = 0$$

$$1-\lambda[(2-\lambda)(3-\lambda)+2[-2(2-\lambda)]] = 0$$

$$1-\lambda[6-2\lambda-3\lambda+\lambda^2] + [-4(2-\lambda)] = 0$$

$$6-2\lambda-3\lambda+\lambda^2 - 6\lambda+2\lambda^2+3\lambda^2-\lambda^3-8+4\lambda=0$$

$$6-8-\lambda^3-7\lambda+6\lambda^2=0$$

$$-\lambda^3+6\lambda^2-7\lambda-2=0$$

$$\lambda^3-6\lambda^2+7\lambda+2=0$$

Character eq.^{n}

Let, $\lambda = 4$

$$A^3-6A^2+7A+2I=0 \quad \text{—(1)}$$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Put in eqn 1

$$A^3 - 6A^2 + 7A + 2I = 0$$

$$\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Hence, Cayley Hamilton therom is verified

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To find A^{-1}

Multiply by A^{-1} in eq ① by both side

$$A^2 - 6A + 7I + 2A^{-1} = 0$$

$$-2A^{-1} = A^2 - 6A + 7I$$

$$-2A^{-1} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 12 \\ 0 & 12 & 6 \\ 12 & 0 & 18 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$-2A^{-1} = \begin{bmatrix} 6 & 0 & -4 \\ 2 & -1 & -1 \\ -4 & 0 & 2 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -3 & 0 & 2 \\ -1 & 1/2 & 1/2 \\ 2 & 0 & -1 \end{bmatrix}}$$

Ans.

Que. 7 $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} = 0$

Verify Cayley Hamilton theoram & find A^{-1}.

For character equation

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 1 \\ 0 & 1-\lambda & -1 \\ 3 & -1 & 1-\lambda \end{vmatrix} = 0$$

$$1-\lambda \begin{vmatrix} 1-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} 0 & -1 \\ 3 & 1-\lambda \end{vmatrix} + 1 \begin{vmatrix} 0 & 1-\lambda \\ 3 & -1 \end{vmatrix} = 0$$

$$1-\lambda \left[(1-\lambda)^2 - 1 \right] - 2(0+3) + 1 \left[0 - 3(1-\lambda) \right] = 0$$

$$1-\lambda \left[1+\lambda^2 - 2\lambda - 1 \right] - 6 - 3 + 3\lambda = 0$$

$$\lambda^2 - 2\lambda - \lambda^3 + 2\lambda^2 - 6 - 3 + 3\lambda = 0$$

$$-\lambda^3 + 3\lambda^2 + \lambda - 9 = 0$$

$$\boxed{\lambda^3 - 3\lambda^2 - \lambda + 9 = 0}$$

Character equation !

Let $\boxed{\lambda = A}$

$$A^3 - 3A^2 - A + 9 = 0 \quad \text{—— (1)}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} = 0$$

$$A^2 = \begin{bmatrix} 4 & 3 & 0 \\ -3 & 2 & -2 \\ 6 & 4 & 5 \end{bmatrix} = 0$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 3 & 0 \\ -3 & 2 & -2 \\ 6 & 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} = 0$$

$$A^3 = \begin{bmatrix} 4 & 11 & 1 \\ -9 & -2 & -7 \\ 21 & 11 & 7 \end{bmatrix} = 0$$

Put in eq. (1)

$$A^3 - 3A^2 - A + 9I = 0$$

$$\begin{bmatrix} 4 & 11 & 1 \\ -9 & -2 & -7 \\ 21 & 11 & 7 \end{bmatrix} - \begin{bmatrix} 12 & 9 & 0 \\ -9 & 6 & -6 \\ 18 & 12 & 15 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Verify Cayley Hamilton Theoram

to find A^{-1},

Multiply by A^{-1} in eq. (1)

$$A^2 - 3A - I + 9A^{-1} = 0$$

$$-9A^{-1} = A^2 - 3A - I$$

$$-9A^{-1} = \begin{bmatrix} 4 & 3 & 0 \\ -3 & 2 & -2 \\ 6 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 6 & 3 \\ 0 & 3 & -3 \\ 9 & -3 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$-9A^{-1} = \begin{bmatrix} 0 & -3 & -3 \\ -3 & -2 & 1 \\ -3 & 7 & 1 \end{bmatrix}$$